

A novel boundary condition for the simulation of the submerged bodies using lattice boltzmann method

**Sh. Sharafatmandjoor; F. Sabetghadam; H. R. Fathalizadeh; A. Shahirpour*

Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

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ABSTRACT: In this study, we proposed a novel scheme for the implementation of the no-slip boundary condition in the lattice Boltzmann method (LBM). In detail, we have substituted the classical bounce-back idea by the direct immersed boundary specification. In this way we construct the equilibrium density functions in such a way that it feels the no-slip boundaries. Therefore, in fact a kind of equilibrium boundary condition is made. Results show that the proposed method presents also a faster solution procedure in comparison to the bounce-back scheme.

Key words: *No-slip Boundary Condition; Submerged Bodies; Lattice-Boltzmann Method; Bounce-Back Technique*

INTRODUCTION

LBM has been successfully employed for the numerical investigation of the hydrodynamics of the incompressible fluid flows as an alternative to the traditional computational fluid dynamics methods (Gladrow, 2005). Since LBM is inspired by the lattice gas cellular automata (LGCA), the bounce-back technique has been the only technique to model the effect of solid boundaries in LBM (Rohde *et al.*, 2003). This scheme is particularly used for the D_nQ_m configurations, although, it seems that it is at most first-order accurate. This maximum accuracy is only achievable in very specific situations (Junk *et al.*, 2005). The reasons could be the deficiency of the limited directional functionality of the scheme. In fact, the reflective rules of the density functions may implement the no-slip boundaries nonphysically.

In order to remedy the shortcomings of the bounce-back scheme, several modifications have

been proposed. For example, modifications include attempts to increase the accuracy of the scheme to two (Rohde *et al.*, 2003), amending the scheme to complex geometries (Verberg *et al.*, 2001) and interpolation technique (Bouzidi *et al.*, 2001; Lee *et al.*, 2006). It should be noted that some non-general successful treatments such as probabilistic or partial bounce-back model have been proposed in areas like porous media, but a unified improvement to the bounce-back technique is still absent.

In this study, inspired by the impressive idea of the immersed boundary methods, we replace the traditional bounce back scheme with the direct implementation of the boundaries that are immersed into to fluid domain. This modification seems to yield more natural, simpler and faster implementation of the no-slip boundary conditions. It also facilitates to explicitly impose well-documented classical computational fluid dynamics boundary conditions such as far-field and free-slip boundary conditions in lattice-Boltzmann simulations.

* Corresponding Author Email: sharafatmandjoor@gmail.com

MATERIALS AND METHODS

Standard Lattice Boltzmann Method

The LBM is a set of rules on the evolution of the probability distribution functions (f) of the particles at locations \mathbf{x} at time t with the discrete lattice velocities along each lattice direction i during the time interval Δt as \mathbf{c}_i . Adopting the single relaxation Bhatnagar–Gross–Krook (BGK) approximation it reads

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = \omega [f_i^{eq} - f_i(\mathbf{x}, t)] \quad (1)$$

where $\omega = \Delta t / \tau$, and τ denotes the relaxation time. f_i^{eq} represents the discretized equilibrium state distribution and is expressed as

$$f_i^{eq} = \omega_i \rho [1 + 3(\mathbf{c}_i \cdot \mathbf{u}) + 9/2(\mathbf{c}_i \cdot \mathbf{u})^2 - 3/2(\mathbf{u} \cdot \mathbf{u})] \quad (2)$$

where for the D_2Q_9 , the weightings are as follows:

$$w_i = 4/9 \text{ for } |\mathbf{c}_i| = 0, w_i = 1/9 \text{ for } |\mathbf{c}_i| = 1 \text{ and } w_i = 1/36 \text{ for } |\mathbf{c}_i| = \sqrt{2}.$$

In these expressions, the flow properties are defined as

$$\mathbf{r} = \sum_i \mathbf{c}_i f_i, \mathbf{r}u = \sum_i \mathbf{c}_i f_i \mathbf{c}_i \quad (3)$$

In a typical simple lattice Boltzmann method, equation 1 splits into two essential steps, namely collision and streaming. Hence, the corresponding computations of LBM are performed as local collision step and non-local streaming steps:

$$\hat{f}_i(\mathbf{x}, t) = \omega [f_i^{eq} - f_i(\mathbf{x}, t)], f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = \omega \hat{f}_i(\mathbf{x}, t) \quad (4)$$

where $\hat{f}_i(\mathbf{x}, t)$ denotes the post-collision state of the distribution function.

In the bounce-back that is the the most commonly used method for simulating no-slip boundary condition the density functions should reflect in the opposite

direction, if they oppose a solid wall.

Implementation of the Immersed Boundary Concept on the Procedure of Lattice Boltzmann Method

The process is as follows. After proper initializations, in the beginning of each step we employ the equilibrium boundary conditions (Izquierdo et al., 2009) for the inlet and outlet. In our test case, equilibrium boundary conditions are applied to inlet and outlet boundaries. Then, we let the distribution function f to stream. In the next step the bounce-back technique calculates the reflected values of f while instead, in the present method we obtain the macroscopic values u, v and ρ from the streamed f . In the next important stage, a window function W that is zero for the solid no-slip boundaries and is unity for the fluid parts is multiplied by u to get u_{IB} . Now we employ the boundary-conditioned velocity field u_{IB} to compute the boundary-conditioned equilibrium distribution function f_{IB}^{eq} . The key point is that f_{IB}^{eq} has sensed the effect of no-slip boundaries. The final stage is the application of the collision operator that uses f_{IB}^{eq} to get the updated value of f .

RESULTS AND DISCUSSION

Now, the well-known problem of the flow over a stationary circular cylinder that is asymmetrically placed in a channel is investigated. Here, in order to assess the capability of the proposed method in dealing with the curved boundaries, we apply the method of (Cheng Chang et al., 2009) to simply update the velocities of the very first node in the flow field adjacent to the physical no-slip boundary by interpolating between the zero velocity physical boundary and the value of the nearest computational node. It should be noted that our method differs from the method of (Cheng Chang et al., 2009) in the way that we do not deal with the probability distribution functions and instead the macroscopic velocity magnitude is directly incorporated in the equilibrium distribution function.

We have reproduced the results of (Cheng Chang *et al.*, 2009) with the same coordinates and values. Fig. 1 shows the solution domain and the fully-developed vortex shedding corresponding to $Re = 100$. Equilibrium boundary conditions are used for the inlet (with parabolic velocity profile) and also for the outlet. The characteristic length in the non-dimensional coefficients is the diameter of the circle D . The pattern of shedding is in agreement with (Cheng Chang *et al.*, 2009). Furthermore, the comparison of coefficients of St , C_l and C_D with other previous results are brought in Table 2.

Now, we investigate how our methodology preserves the continuity principle. Therefore, we have showed the normalized second norm of the residual

$R = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ over the flow domain in Fig. 2 for different cylinder diameters. The results show that the method is convergent in the sense that if we increase the resolution the residual decreases as D_p , where $1.9 \leq P \leq 2.2$.

Note that the solution with the proposed method is not fundamentally sensitive to the interpolation scheme. In fact, combining the immersed boundary techniques with LBM is superior to combining it with other higher-order methods *e.g.* spectral and spectral element methods. In such methods, the Gibbs phenomenon created by the sharp interfaces is hardly removed by the interpolation itself (Boyd, 2005). On the other hand, the order of accuracy is then controlled by the accuracy of the interpolation which is not comparable to that of the original methods. Fig. 3 illustrates the vorticity patterns with and without the velocity treatment, respectively. By comparison, it can be seen that even with the zeroth-order velocity correction which introduces the circle as a saw-tooth shape, the no-slip condition on the circle is sensed by the flow field. In the figures we have intentionally used low resolutions to show that the feature is independent of the resolution.

Now, to demonstrate the abilities of the proposed method in dealing with the hydrodynamics of the submerged bodies, a typical inclined airfoil is faced

with inlet flow in a channel where $Re = 100$. This implementation is a simple extension of the cylinder problem, where the circle is replaced by an airfoil. Fig. 4 shows the vorticity field contours. The inclination angle causes the vortical structures to shed behind the airfoil which is an expected phenomenon.

CONCLUSION

The results of this paper prove that using IBM can be a reliable and efficient alternative to the traditional bounce-back no-slip boundary implementation in LBM. Besides, the need for avoiding the sharpness of edges which is regarded as a significant constraint in applying IBM to finite difference and spectral approaches, will contribute to merely improving the results and is not regarded as a limit. In addition, the proposed method provides saving in computational requirements with respect to the classical bounce-back method. Although, dealing with no-slip boundary conditions is the main aim of this paper, according to existing experience in applying IBM, the method is also applicable for applying general Dirichlet boundary conditions with non-zero velocities to LBM.



Fig. 1: Fully developed vorticity field for flow over an asymmetrically placed cylinder in a channel

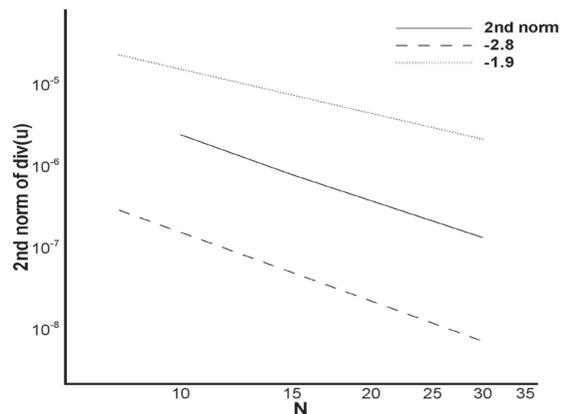


Fig. 2: Second Norm of the Residual Value

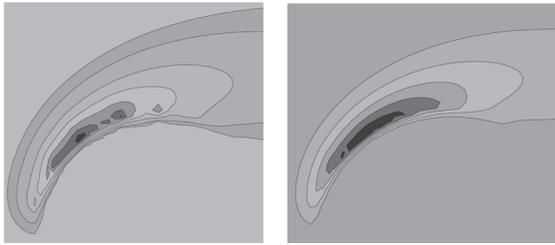


Fig. 3: Vorticity Field for the Cylinder , without Treatment(Left) and with Treatment(Right).



Fig. 4: Vorticity Contours of the Flow over the Inclined Airfoil in a Channel

Table 2: Comparison of lift coefficient, drag Coefficient and Strouhal

	Chen et al.	Schäfer and Turek	Present work
St	0.302	0.300	0.3069
C_D	3.33	3.22	3.27-3.41
C_l	1.051	0.99	0.9927

Number for $Re=100$

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