3D Finite element modeling for Dynamic Behavior Evaluation of Marin Risers Due to VIV and Internal Flow

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**ABSTRACT:** The complete 3D nonlinear dynamic problem of extensible, flexible risers conveying fluid is considered. For describing the dynamics of the system, the Newtonian derivation procedure is followed. The velocity field inside the pipe formulated using hydrostatic and Bernoulli equations. The hydrodynamic effects of external fluids are taken into consideration through the nonlinear drag forces in various time steps and the added inertia due to the hydrodynamic mass. Following the Newtonian derivation, the dynamics of the pipes element with effects of internal fluid are considered separately and the final governing set is derived by combining the equations of inertia equilibrium. The study focuses specifically on the effect of the inner flow to the global dynamics of the riser. This task is accomplished using time domain teachings and the Finite Element method is used as a powerful numerical method. Moreover the Euler-Bernoulli beam theory is used response to model the dynamic behavior of the flexible risers.

**Keywords:** risers, inner flow, dynamic response, Finite Element Method (FEM)

**INTRODUCTION**

The dynamic behavior of cylindrical pipes with axial flow has been extensively investigation in the past (Paidoussis, 1998; Paidoussis, 2001). There is an enormous amount of works reported in the literature that treat a bulk of important engineering application. An interesting subject in this context is the application of flexible pipes that transport fluids, especially when they operate in a wet environment and they are subjected to external motions and hydrodynamic forces. (Paidoussis, 2005)

Indicative examples of studies on the dynamics of flexible Risers transporting fluid are those due to (Misra, et al, 1988; Jain and Jayaraman, 1990; Dupuis and Rousselet, 1992; Semler, et al, 1994; Petrakis and Karahalios, 1997; Qiao et al, 2006; Lin et al, 2007). The investigation concerns mainly 2D formulation, while there are also studies that examine problems in 3D, such as the work reported by (Tanjugu et al, 2007) who investigated the out-of-plane vibrations of flexible risers due to pulsating flow and (Chai and Varyani, 2006) who presented a general absolute coordinate formulation for the 3D analysis of flexible pipe structure including the effect of internal flow. The authors used a similar line of approach adopted by (Pauling and Webster, 1986).

Recently, (Wadham-Gagnon, 2007) developed the nonlinear equation of 3D motion for unrestrained and restrained cantilevered pipes conveying fluid. For the unrestrained pipes, the equation of motions where derived by assuming that the fluid is incompressible, the flow is of constant velocity and the pipe behaves as a nonlinear Euler-Bernoulli beam. Moreover, the strain of the pipe was considered small while the rotary inertia and the shear deformation were neglected.

Typically, marine risers or free span pipe lines are lengthy structures, subjected to externally imposed excitations of various amplitudes and frequencies and they are characterized by a relatively small equivalent elasticity $\frac{EA}{L}$ since their large size. In addition, instability phenomena are difficult to be detected due to the drastic contribution of drag forces. Furthermore, the treatment of the problem in tow dimensions is admittedly a short approximation as these structures can be subjected to high frequency cycling motions due to the externally imposed excitations that originate from the behavior is primarily governed by the imposed motions which are normally applied at one end. Never the less, it is very interesting to investigate the details of the 3D dynamic of the associated nonlinear system under the combined contribution of both sources of excitation, namely external motions and internal flow. (Lecunff and Biloey, 2005; Chatjigeorgiou and Damy, 2007)
Most of the studies on the dynamic of risers, at least at they reviewed by (Chakrabarti and Frampton, 1982; Jain, 1994; Patel and Seyed, 1995) omit the contribution of the internal flow. Moreover, with regard to the works that consider the effect of the fluid's velocity inside the pipe, it appears that special attention is given to 2D formulation. Relevant examples are the works of (Wu and Lou, 1991; Bar-Avi, 2000; Chucheepsakul et al, 2003; kuiper et al, 2004; Chatjigergiou and Mavrakos, 2005; Kaewunruen et al, 2005; Kupier and Metrikine, 2005; Monprapussorn et al, 2007; Kuiper et al, 2008).

There are also studies special effects induced by internal flow such as Vortex-induced-vibration or Whipping phenomena. (Bordalo et al, 2008)

The present paper is dedicated to the formulation and the solution of the complete 3D dynamic problem of a flexible riser including the effect of the internal flow. Effort has been made in developing a generic formulation that could describe the dynamics of general shapes of flexible risers or free span pipelines. The work extends the research of the present author on nonlinear dynamics of flexible risers using an efficient finite element methods. (Chatjigeorgiou et al, 2008) This is achieved by extending the existing 2D formulation to 3D and incorporating the internal flow.

The dynamic problem is also treated in the time domain. The main goal in this context is the derivation of the transfer functions of all dynamic components, both in-plane and out-of-plane, that influence the dynamics of the structures. The results from the solution in the time domain is of paramount importance as the transfer functions of the concerned variables can easily demonstrate the details of the dynamic behavior of the structure subjected to the effect of the internal flow and the impacts of forced excitations, while in addition this can be done with a very descriptive way.

**MATERIALS AND METHODS**

**Mathematical formulation and governing differential equations**

In this section the mathematical formulations for a vertical pin-end beam subjected to the varying tensions to the top tensions and the internal flow effects and the impacts of hydrodynamic forced due to the effects of the external fluid are explained.

In beams with high aspect ratio such as riser pipes, Euler-Bernoulli beam theory can be used to model the dynamics, because the transverse shear can be neglected. When the diameter or width of the cross-section is small compared to the length of the beam, it can be considered that the planes perpendicular of the axis remain plane and perpendicular to the axis after deformation. (HueraHuarte, 2006)

The system of dynamic equilibrium is formulated under the following assumptions: (i) the flow inside the pipe is inviscous, incompressible and irrotational; (ii) the pipe behaves as a nonlinear Euler-Bernoulli beam; (iii) the centerline of the pipe is extensible obeying to a liner stress-strain relations; (iv) shear, bending and torsion effects are incorporated in to the mathematical model but the final equations are solved omitting torsion; (v) the ends of the pipe are pinned; (vi) planar surfaces orthogonal to the axis of the beam, remain planar and orthogonal to the axis after the deformation.

A cartesian reference with its origin at the bottom of the riser has been used, in which the X axis is parallel to the external flow velocity, Z coincides with the vertical axis of the riser in its undeflected configuration and the Y is perpendicular to both. u(z, t), v(z, t) and w(z, t) are defined as the time variant in-line, cross-flow and the axial motions respectively. With these set of displacements a point in the centerline of the beam can be spatially described.

Combinations of translations and rotations around the beam axis describe states of torsion, and the states of bending are described by displacements and rotations around the two axes contained in the plane perpendicular to the beam axis. Because the riser was attached to the supporting structures at its ends with universal joints, torsion motions were avoided, but the fact that the riser model was free to move in-line and cross-flow at the same time, meant that small twisting motions were inevitable. Moreover when the first derivative in space of the transverse motions is small \( \frac{\partial u}{\partial z} \approx 0 \) and \( \frac{\partial v}{\partial z} \approx 0 \), as in the case presented here, the non-linear terms in the stress equations disappear and both transverse and axial motions become uncoupled. (Reddy, 1993) and (Reddy, 2005).

Therefore, in problems involving small displacement the motions in the planes XZ and YZ can be treated independently.

**Equation for the transverse motion (X or Y directions)**

The transverse deformation of a generic beam can be described with a fourth order differential equation which can be derived by applying force and momentum equilibrium to an infinitesimal section of the beam, see the free-body diagram in Figure (1).

The formulation can be similarly applied to any of the tow transverse directions, X or Y, where \( f \) is the external fluid force, \( \dot{f} \) is the inertia force and \( \tau \) is the shear force, all referred to one of the transverse directions, X or Y.
we obtain the differential equation that governs the transverse of a beam with flexural stiffness $EI(z)$ and with an applied tension $T(z)$. This equation has been obtained neglecting the effects of rotational inertia and damping (HueraHuarte, 2006).

$$\frac{d^4}{dz^4}(EI(z)\frac{d^2u(z,t)}{dz^2}) - \frac{d}{dz}(T(z)\frac{du(z,t)}{dz}) + m(z)\frac{d^2u(z,t)}{dt^2} = f(z,t)$$

The boundary conditions for the above differential equation in the case of a pin-ended beam are:

$$u(0,t) = 0, \quad u(L,t) = 0 \quad \forall t$$

$$\frac{d^2u(0,t)}{dz^2} = 0, \quad \frac{d^2u(L,t)}{dz^2} = 0 \quad \forall t$$

The equation above includes the effect of axial tension and can be used to describe the deflection of the riser model used for the experiments. This equation is a fourth order partial differential equation with a time variant term, and it can be rewritten as:

$$EI \frac{d^4u(z,t)}{dz^4} - \frac{d}{dz}(T(z)\frac{du(z,t)}{dz}) + m \frac{d^2u(z,t)}{dt^2} = f(z,t)$$

Assuming the mass $m(z)=m$ and the flexural stiffness $EI(z)=EI$ are uniform along the length of the riser due to the design of the model. The tension in equation 12 can be expressed as:

$$T(z) = T_t - \omega_z(L - z)$$

with $T_t$ being the tension applied at the top of the riser, $L$ the length of the riser and $\omega_z$ is the submerged weight per unit length. The equation consider the effect of buoyancy on the riser because the submerged weight is used and considered to be constant. In addition, the hydrodynamic pressure due to the inner circulating fluid has an effect on the effective tension variation along the length of riser.

**Equation for the axial motion (Z direction)**

Applying the same assumptions as in section 2.1 the equations for the axial motion can be derived. The definition of strain allows us to derive the expression of the tension relating it to the axial motions. The general expression for the axial strain can be found in (Reddy, 2005). In the present work, it will be referred to from now on as $\varepsilon$, being:

$$\varepsilon = \frac{\partial w}{\partial z} + \frac{1}{2}\left(\frac{\partial u}{\partial z}\right)^2$$

If there are not large displacements, geometric non-linearity coming from these terms can be neglected because $\frac{\partial u}{\partial z} \approx 0$ and $\frac{\partial w}{\partial z} \approx 0$. Therefore, $\varepsilon$ becomes:

$$\varepsilon(z,t) = \frac{\partial w(z,t)}{\partial z}$$

The stress is related to the strain by means of the Elastic modulus.

$$\sigma(z,t) = E\varepsilon(z,t) = E \frac{\partial w(z,t)}{\partial z}$$

and the tension is related to the stress through the cross-sectional area of the beam.

$$T(z,t) = A\sigma(z,t) = EA \frac{\partial w(z,t)}{\partial z}$$

In this case, because of its relation to the axial strain, the tension is a function not only of $Z$ but also of time (HueraHuarte, 2006).

Hence, the equation for the axial motion of the beam is

$$m \frac{d^2w(z,t)}{dt^2} - EA \frac{d^2w(z,t)}{dz^2} = f_z(z,t)$$

with the following boundary conditions:

$$w(0,t) = 0 \quad \forall t$$

**RESULTS AND DISCUSSION**

The finite element method (FEM)

The finite element method is based on following steps:

1. Discretisation of the domain of the governing equation into small parts called Finite Elements.
2. Transformation of the governing equation in an integral form (weak formulation) because integral equations are easier to solve numerically.
3. Approximation of the solution as a linear combination of special function in each element of the domain and assembly of the approximations for each element to obtain the global system.
4. Time discretization.
After these steps the initial partial differential equation are transformed into an algebraic system and much easier to solve.

**Discretization of the riser**

The flexible riser is idealized as a one-dimensional domain with the neutral axis of the structure. A mesh formed by one-dimensional elements can be used, in which a generic finite element $Ω^e$ consists of tow nodes, and is referred to, as

$$Ω^e = [z_1^e, z_2^e]$$

with the length of element defined as

$$h^e = z_2^e - z_1^e$$

being $z_1^e$ and $z_2^e$ the global coordinate of the finite element at the first and the second node respectively. The number of element, $n_e$ is

$$n_e = \frac{L}{h^e}$$

and the number of nodes

$$n = n_e + 1$$

The solution of the equation 3 in each element will be approximated with

$$u^e(z, t) ≃ u^e(t)Ψ^e(z) = \sum_{j=1}^{N_t} u_j^e(t)ψ_j^e(z)$$

$$v^e(z, t) ≃ V^e(t)Ψ^e(z) = \sum_{j=1}^{N_t} v_j^e(t)ψ_j^e(z)$$

$$w^e(z, t) ≃ W^e(t)L^e(z) = \sum_{j=1}^{N_t} w_j^e(t)l_j^e(z)$$

According to this approximation, the transverse deflections of the riser in each finite element, depending on time and position, $u(z, t)$ and $v(z, t)$, are the linear combination of the time dependent deflections at the node $j$ that is $u_j^e(t)$ and $v_j^e(t)$, multiplied by spatial approximation functions $ψ_j^e(t)$, according to each of the $N_t$ degrees of freedom in both nodes of the finite element. The same applies of the axial motions $ω_j^e(t)$ with the spatial functions $l_j^e(t)$s and $N_a$ degree of freedom. The approximation function are known as shape functions.

The bending states result from motions and rotations (first derivatives in space of the displacements), so the finite elements for the transverse case must be able to represent tow degrees of freedom at each node, and it implies $N_t=4$ degrees of freedom. The rotations, around the axis of the pipe are neglected, and this result in only one degree of freedom at each nodes of the finite element associated to the axial displacements with $N_a=2$ degree of freedom.

**Weak formulation**

The need for an integral form of equation 3 comes from the fact that if equations 15 and 16 are substituted in the governing equation 3, it is possible that the resultant system would not always have the required number of linearly independent algebraic equations needed to find the coefficients $U_j^e(t)$, $V_j^e(t)$, that represent the solution of our system at each node. The same could happen with $W_j^e(t)$ when substituting eq.17 in the axial equation of motion, eq.9. A way to obtain the correct number of linearly independent equations is to final the integral weak formulation of the governing equation (Reddy, 1993). A weight function $υ(z)$ is used for the purpose, multiplying the governing equation and integrating along the element to find the weak form of the differential equation.

**Weak formulation for transverse equations of motion (X or Y direction)**

In the case of the transverse equations of motion the procedure is valid for both the equations modeling the motion in the XZ and YX planes. Introducing the weight function $υ(z)$ into equation 3,

$$\int_{z_1^e}^{z_2^e} \partial_z \partial_τ θ(z) \left[ EI \frac{∂^2 u(z, τ)}{∂z^2} - \frac{∂}{∂z} \left( T(z) \frac{∂u(z, τ)}{∂z} \right) + m \frac{∂^2 u(z, τ)}{∂τ^2} - f(z, τ) \right] dz = 0$$

Integration by parts once in the second order term, and twice in the fourth order one, allows to obtain the weak formulation of the differential equation as follow:

$$\int_{z_1^e}^{z_2^e} T(z) \frac{∂θ}{∂z} \frac{∂u}{∂z} + EI \frac{∂υ}{∂z} \frac{∂^2 u}{∂z^2} + mw \frac{∂^2 u}{∂τ^2} - w f(z, τ) \partial_z \partial_τ θ(z) = 0$$

Because it is a fourth order differential equation we have two primary variables $u(z,t)$ and $w_τ(z,t)$. That means each nodes has associated with it two degrees of freedom, one for each primary variable on the other hand, the secondary variables are:

$$Q_1(τ) = \left[ -T(z) \frac{∂u}{∂z} + \frac{∂}{∂z} \left( EI \frac{∂^2 u}{∂z^2} \right) \right]_{z = z_1^e}^{z = z_2^e}$$

$$Q_2(τ) = - \left[ -T(z) \frac{∂u}{∂z} + \frac{∂}{∂z} \left( EI \frac{∂^2 u}{∂z^2} \right) \right]_{z = z_1^e}^{z = z_2^e}$$

$$Q_1(t) = EI \frac{∂υ}{∂z} \bigg|_{z = z_1^e}^{z = z_2^e}$$

$$Q_2(t) = EI \frac{∂υ}{∂z} \bigg|_{z = z_1^e}^{z = z_2^e}$$

Where the first ones represent shear forces and the second ones bending moments. equation 24 results in
a system of algebraic equations that gives the approximate solution of 3 in the generic element $\Omega^e$. It is the algebraic equation for both transverse motions, either X or Y. It can be expressed in matrix form as:

$$K^e U^e + M^e \ddot{U}^e = F^e_x + Q^e_x \quad \text{In-line} \quad (24)$$

$$K^e V^e + M^e \ddot{V}^e = F^e_y + Q^e_y \quad \text{Cross-flow} \quad (25)$$

where the vectors $U^e$ and $V^e$ gives the displacements along the axis of the cylinder in the transverse directions. The matrices are $[4^e4]$ because there are two nodes and at each of them, two degrees of freedom. Note that the displacements vector $U^e$ is formed by $u_1$ and $\frac{\partial u}{\partial z}$ at each node, then its components are $U^e=[u_1 \ u_2 \ u_3 \ u_4]^T$. $u_1$ is the displacement at the first node ($Z_1^e$), $u_2 = -\frac{\partial u}{\partial z}$ is the rotation in the first node. $u_3$ is the displacement at the second node ($Z_2^e$) and $u_4 = -\frac{\partial u}{\partial z}$ the rotation at the second node. $K^e$ is the stiffness matrix, $M^e$ is the consistent mass matrix $Fe$ is the transverse nodal forces vector, and finally $Q^e$ is the secondary variables vector. The reader must notice that $K^e$ and $M^e$ are valid for any of the two transverse directions, X or Y. They are both symmetric and calculated as follows,

$$K^e = K^e_1 + K^e_2$$

$$K^e = \begin{bmatrix} K^e_{11} & K^e_{12} \\ K^e_{21} & K^e_{22} \end{bmatrix}$$

$$K^e_{11} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}, K^e_{12} = \begin{bmatrix} K_{13} & K_{14} \\ K_{23} & K_{24} \end{bmatrix}, K^e_{22} = \begin{bmatrix} K_{31} & K_{32} \\ K_{41} & K_{42} \end{bmatrix}$$

$$K^e_1 = \begin{bmatrix} K_{51} & K_{52} \\ K_{61} & K_{62} \end{bmatrix}, K^e_2 = \begin{bmatrix} K_{53} & K_{54} \\ K_{63} & K_{64} \end{bmatrix}$$

(27)

Taking into account that the same nomenclature is followed for the mass matrix $M^e$.

**Weak formulation for axial equations of motion (Z direction)**

The procedure of the case of the axial equation of motion is the same. The weight functions are introduced in equation follow multiplying all its terms and then the integration is developed.

$$\int_{z_1^e}^{z_2^e} \varphi(z) \left[ E I \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\partial}{\partial z} \left( T(z) \frac{\partial u(z,t)}{\partial z} \right) + m \frac{\partial^2 u(z,t)}{\partial z^2} - f(z,t) \right] dz = 0$$

(29)

Integration by parts once in the second order term, and twice in the fourth order one, allows to obtain the weak formulation of the differential equation.

$$\int_{z_1^e}^{z_2^e} \left[ T(z) \frac{\partial^3 u(z,t)}{\partial z^3} + EI \frac{\partial^3 u(z,t)}{\partial z^3} + m \frac{\partial^3 u(z,t)}{\partial z^3} - w f(z,t) \right] dz = 0$$

(30)

where

$$Q_1^e(t) = EA \frac{\partial u}{\partial z} \mid_{x=x_1^e}, \ \ \ \ Q_2^e(t) = EA \frac{\partial u}{\partial z} \mid_{x=x_2^e}$$

(31)

$$Q_3^e(t) = E A \frac{\partial^2 u}{\partial z^2} \mid_{x=x_1^e}, \ \ \ \ Q_4^e(t) = E A \frac{\partial^2 u}{\partial z^2} \mid_{x=x_2^e}$$

(32)

Where the first ones represent shear and the second ones bending moments.

The equations can be written in matrix form as follows:

$$K^e_z W^e + M^e_z \ddot{W}^e = F^e_z + Q^e_z$$

(33)

Where $W^e = [w_1 \ w_2]^T$, with its components the displacements at the first ($Z_1^e$) and second node ($Z_2^e$). Because $EA$, and $m$ are constant along the length of the riser model, they do not vary for the different elements composing the mesh and are easy to solve analytically, result in,

$$K^e_z = \frac{E A}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, M^e_z = \frac{m h}{6} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, F^e_z = \frac{h^2 f}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(34)

**Approximation of the solution as a linear combination of special function in each element of the domain and assembly of the approximations for each element to obtain the global system.**

Once the solution is approximated in a generic element $\Omega^e$, we have to extrapolate the solution to the whole mesh in order to obtain the assembled system that gives the approximate solution for the complete model. The assembly of the equations is based on tow concept:

1. Continuity of the primary variables.
2. Equilibrium of the secondary variables.

The first concept is directly solved from local notation to global notation, this means the values of the variables at the second node of element $\Omega^{n-1}$ must be equal to the value of the variables at the first node of the $\Omega^n$ element. Then if the local notation for the second node of element $\Omega^n$ was $z_{n-1}$ it now becomes $z_n$, the same for the first node of $\Omega^n$ that was before $z_0$. This scheme is valid for $K$ and $M$, and allows us to have the global system of equations that give approximate solution of equations 3 and 9 for the riser model.
\[
K = \begin{bmatrix}
K_1 & K_2 & 0 & \cdots & \cdots & 0 \\
K_3 & (K_1 + K_2) & K_4 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\
0 & \cdots & K_2 & (K_1 + K_2) & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \vdots & \vdots \\
0 & \cdots & \cdots & 0 & K_2 & (K_1 + K_2) \\
0 & \cdots & \cdots & \cdots & 0 & K_2
\end{bmatrix}
\]

\[K.r + M.\ddot{r} = F_Q + Q = F\]  \hspace{1cm} (35)

\[Q_2 \epsilon + Q_1 \epsilon^{n+1} = 0\]  \hspace{1cm} (36)

The effect of the flow inside the pipe can be accurately represented by the simplified flow model. To support this statement calculation, have been performed with the contribution of the perturbation component of the velocity field \(\phi\) and the associated results are depict in Fig. 4 These figures show respectively the variation of tension \(T\), for different quantities of internal flow velocities, the in-line bending moment and the out-of-plane bending moment.

Fig. 5 and 6 show the response of the structure for limited time domain under the velocity hydrodynamic forces. In Fig. 5 the curves show the amount of deflections. And finally Fig. 7 and 8 shows the variation of the in-plane and out-of-plane bending moment along the structure for various internal flow velocities. In fact this curves are the results for solutions of differential Equations.
Fig. 3: show snapshots of the axial force $F$ due to the internal pressure

Fig. 4: Snapshots of the variation of tension (T) along the structure due to the variation of internal fluid velocity
Fig. 5: Snapshots of the variations of the response the structure for the in-plane-direction

Fig. 6: Snapshots of the variations of the response the structure for the Out-of-plane-direction
CONCLUSION
The non linear dynamic behaviour of a submerged, extensible flexible riser conveying internal fluid and subjected to hydrodynamic force was considered. The problem was solved using time domain solution techniques.
The formulations of the dynamic systems that correspond to the pipe and the fluid elements are combined to yield a single system that concerned the dynamic of the curved riser incorporating terms that arise from the dynamics of the fluid, namely the pressure and the velocity inside the pipe.
It was shown that the additional axial force due to the dynamic pressure distribution inside the pipe is calculated and the pressure due to the velocity of the steady flow is considered.
The solution of the non linear problem in the time domain demonstrated that only the out-of-plane response is affected by the internal flow, the validity of the observation was verified also by the other
numerical solution, that name is finite difference method. It was shown that the Coriolis forces which are incorporated into the dynamic system of the riser due to the steady velocity term affect only the out-of-plane motions reducing the magnitudes of the associated dynamic components along the complete length of the structure.

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